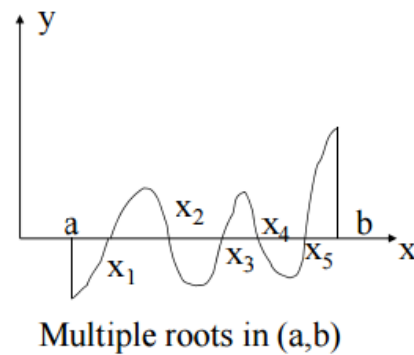
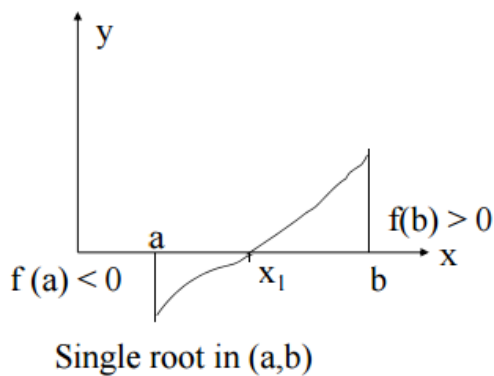


# Root Finding: $f(x)=0$

## Method 1: The Bisection method

Theorem: If  $f(x)$  is continuous in  $[a,b]$  and if  $f(a)f(b)<0$ , then there is atleast one root of  $f(x)=0$  in  $(a,b)$ .



Example:

Consider finding the root of  $f(x) = x^2 - 3$ . Let  $\epsilon_{\text{step}} = 0.01$ ,  $\epsilon_{\text{abs}} = 0.01$  and start with the interval  $[1, 2]$ .

Bisection method applied to  $f(x) = x^2 - 3$ .

| A       | b      | f(a)    | f(b)   | $c = (a + b)/2$ | f(f)    | Update | b - a  |
|---------|--------|---------|--------|-----------------|---------|--------|--------|
| 1       | 2      | -2      | 1      | 1.5             | -0.75   | a = c  | 0.5    |
| 1.5     | 2      | -0.75   | 1      | 1.75            | 0.062   | b = c  | 0.25   |
| 1.5     | 1.75   | -0.75   | 0.0625 | 1.625           | -0.359  | a = c  | 0.125  |
| 1.625   | 1.75   | -0.3594 | 0.0625 | 1.6875          | -0.1523 | a = c  | 0.0625 |
| 1.6875  | 1.75   | -0.1523 | 0.0625 | 1.7188          | -0.0457 | a = c  | 0.0313 |
| 1.7188  | 1.75   | -0.0457 | 0.0625 | 1.7344          | 0.0081  | b = c  | 0.0156 |
| 1.71988 | 1.7344 | -0.0457 | 0.0081 | 1.7266          | -0.0189 | a = c  | 0.0078 |